PRINT Your Name:___________________________________

Quiz 3, Spring, 2013

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner.

Calculate $\iint_R (x+y)^2 e^{x-y} dxdy$, where R is the region bounded by $x+y=1$, $x+y=4$ $x-y=-1$ and $x-y=1$.

Answer: Observe that R is the rectangle with vertices $(1,0)$, $(\frac{5}{2},\frac{3}{2})$, $(\frac{3}{2},\frac{5}{2})$, $(0,1)$. We create a transformation T from the the unit square (call this U) to R in two steps. First we send the unit square to the rectangle with vertices $(0,0)$, $(\frac{3}{2},\frac{3}{2})$, $(\frac{1}{2},\frac{5}{2})$, $(-1,1)$. Then we add 1 to each x-coordinate. (I have drawn a picture on a separate page.) So

$$\begin{bmatrix} x \\ y \end{bmatrix} = T\left(\begin{bmatrix} u \\ v \end{bmatrix}\right) = \begin{bmatrix} \frac{3}{2} & -1 \\ \frac{3}{2} & 1 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

In other words,

$$x = \tfrac{3}{2}u - v + 1 \quad \text{and} \quad y = \tfrac{3}{2}u + v.$$

Do notice that

$$(0,0) \mapsto (1,0), \quad (1,0) \mapsto (\tfrac{5}{2},\tfrac{3}{2}), \quad (1,1) \mapsto (\tfrac{3}{2},\tfrac{5}{2}) \quad (0,1) \mapsto (0,1),$$

as desired. The Jacobian is

$$\left|\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}\right| = \left|\begin{vmatrix} \frac{3}{2} & -1 \\ \frac{3}{2} & 1 \end{vmatrix}\right| = 3.$$

Let $f(x,y) = (x+y)^2 e^{x-y}$. The original integral is equal to

$$\iint_U \text{Jac} f(x(u,v), y(u,v)) dudv = 3\int_0^1 \int_0^1 (3u+1)^2 e^{-2v+1} dudv$$

$$= \left.\frac{(3u+1)^3}{3}\right|_0^1 \left.\frac{e^{-2v+1}}{-2}\right|_0^1 = \boxed{\frac{21(e-\frac{1}{e})}{2}}.$$