PRINT Your Name:____

Quiz 2, Spring, 2013

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. The solution will be posted later today.

Let $f(x)$ be a continuous function for $a \le x \le b$.

(a) Show that $(\int_a^b f(x)dx)^2$ is equal to $\alpha \int_a^b \int_x^b f(x)f(y)dydx$ for some constant α.

(b) Identify α.

Explain what you are doing very carefully.

Answer: We see that

$$(\int_a^b f(x)dx)^2 =_1 (\int_a^b f(x)dx)(\int_a^b f(x)dx) =_2 (\int_a^b f(x)dx)(\int_a^b f(y)dy)$$

$$=_3 (\int_a^b f(x)(\int_a^b f(y)dy)dx) =_4 (\int_a^b(\int_a^b f(x)f(y)dy)dx)$$

$$=_5 \int\int_{[a,b]\times[a,b]} f(x)f(y)dA.$$

The equalities 1 and 2 are obvious. For equality 3, it is legal to move the constant $\int_a^b f(y)dy$ inside the integral $\int_a^b f(x)dx$. For equality 4, as far as the integral $\int_a^b f(y)dy$ is concerned, $f(x)$ is a constant. It is legal to move the constant inside the integral sign. The left side of equality 4 is an iterated integral; the right side is the corresponding double integral. We split the rectangle $[a,b]\times[a,b]$ into two triangles by drawing the line connecting the corner (a,a) to the corner (b,b).

$$=_6 \begin{cases} \int\int_{\text{the triangle with vertices (a,a),(a,b),(b,b)}} f(x)f(y)dA \\ +\int\int_{\text{the triangle with vertices (a,a),(b,a),(b,b)}} f(x)f(y)dA \end{cases}$$

We fill up the triangle of the first integral using vertical lines. We fill up the triangle of the second integral using horizontal lines.

$$=_7 \int_a^b \int_x^b f(x)f(y)dydx + \int_a^b \int_y^b f(x)f(y)dxdy$$

$$=_8 \int_a^b \int_x^b f(x)f(y)dydx + \int_a^b \int_x^b f(y)f(x)dydx = 2\int_a^b \int_x^b f(x)f(y)dydx.$$

In 8, we replaced all the x's by y's and all of the y's by x's in the second integral.

We have shown that

$$\boxed{(\int_a^b f(x)dx)^2 = 2\int_a^b \int_x^b f(x)f(y)dydx}$$