

A lumberjack cuts out a wedge-shaped piece  $W$  of a cylindrical tree of radius  $r$  obtained by making two saw cuts to the tree's center, one horizontally and one at an angle  $\theta$ . Compute the volume of the wedge  $W$  using Cavalieri's principle. (See Figure 5.1.12.)

# Quiz!

The solution will be posted later today

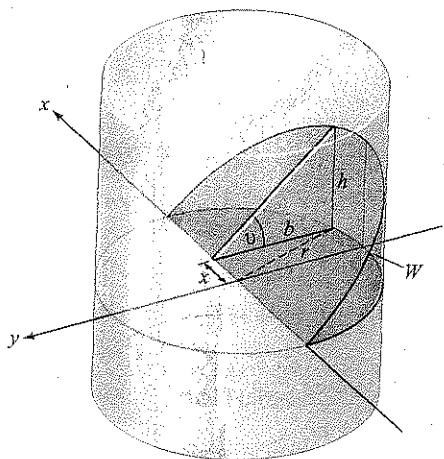
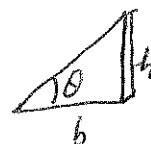


figure 5.1.12 Find the volume of  $W$ .

The volume of the slice that is drawn above is

$\frac{1}{2} b h t$ , where  $t = \Delta x$ ,  $h = b \tan \theta$ , and



so  $b = \sqrt{r^2 - x^2}$

the volume of the slice is

$$\frac{1}{2} b h t = \frac{1}{2} b b \tan \theta t = \frac{1}{2} b^2 \tan \theta \Delta x = \frac{1}{2} (r^2 - x^2) \tan \theta \Delta x$$

The volume of the wedge is obtained by adding up the volume of the slices and taking the limit

$$= 2 \frac{1}{2} \tan \theta \left( r^2 x - \frac{x^3}{3} \right) \Big|_0^r$$

$$= \tan \theta \left( r^3 - \frac{r^3}{3} \right) = \boxed{\frac{2(\tan \theta) r^3}{3}}$$