Math 550, Final Exam, Spring 2013

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. Write coherent correct explanations. Write in complete sentences. $\boxed{CIRCLE}$ your answer, when this makes sense. **No Calculators or Cell phones.**

1. (12 points) Compute $\int_0^1 \int_x^1 e^{y^2}\, dy\, dx$. Explain thoroughly.

2. (11 points) Compute $\frac{d}{dx} \int_a^x \int_c^d f(x,y,z)\, dz\, dy$. Explain thoroughly.

3. (11 points) Find the volume of the region bounded by $z = x^2 + y^2$ and $z = 10 - x^2 - 2y^2$. Explain thoroughly.

4. (11 points) Let $D^* = [0,1] \times [0,1]$ and define T on D^* by $T(u,v) = (-u^2 + 4u, v)$. Find the image of T . Explain thoroughly.

5. (11 points) Let D be the parallelogram with vertices $(1,2)$, $(2,3)$, $(4,6)$, and $(3,5)$. Compute $\int\int_D x\, dx\, dy$. Explain thoroughly.

6. (11 points) Compute the path integral of $f(x,y,z) = xyz$ over the path $c(t) = (\cos t, \sin t, t)$ for $0 \le t \le \frac{\pi}{2}$. Explain thoroughly.

7. (11 points) Consider the force field $\boldsymbol{F}(x,y,z) = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$. Compute the work done in moving a particle along the parabola $y = x^2$, $z = 0$, from $x = -1$ to $x = 2$. Explain thoroughly.

8. (11 points) Find the area of the part of the unit sphere that satisifies $z \ge \sqrt{x^2 + y^2}$. Explain thoroughly.

9. (11 points) Let the velocity field of a fluid be described by $\boldsymbol{F} = \boldsymbol{i} + x\boldsymbol{j} + z\boldsymbol{k}$ (measured in meters per second). Compute how many cubic meters of fluid per second are crossing the surface described by $x^2 + y^2 + z^2 = 1$ with $z \ge 0$. Explain thoroughly.