Math 550, Exam 1, Spring 2013

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

The solutions will be posted later today.

1. (9 points) Compute $\int_0^1 \int_y^1 \sin(x^2)dxdy$. **Explain very carefully what you are doing.**

2. (9 points) Let $f(x)$ be a continuous function for $a \le x \le b$. Find a formula which relates $(\int_a^b f(x)dx)^2$ and $\int_a^b \int_x^b f(x)f(y)dydx$. **Explain why your formula is correct very carefully.**

3. (8 points) A lumberjack cuts a wedge-shaped piece W out of a cylindrical tree of radius a by making two saw cuts. The first cut is parallel to the ground. The second cut makes an angle θ with the first cut and meets the first cut along a diagonal of the circle that contains the first cut. Find the volume of W. **Explain very carefully what you are doing.**

4. (8 points) Let $f(x,y,z)$ be a continuous function which is defined on all of three space. Let a, b, and c be constants. Consider the function $F(x) = \int_c^x \int_a^b f(x,y,z)dydz$. Find an expression for $\frac{d}{dx}F(x)$ in which all differentiation is done before all integration. **Explain very carefully what you are doing.**

5. (8 points) Find a linear map $L\colon \mathbb{R}^2 \to \mathbb{R}^2$ which carries the parallelogram with vertices $(0,0)$, (a,b), (c,d), $(a+c,b+d)$ to the parallelogram with vertices $(0,0)$, (e,f), (g,h), $(e+g,f+h)$. (You may assume that both parallelograms are honest-to-goodness parallelograms.) **Explain very carefully what you are doing.**

6. (8 points) What is the area of the parallelogram with vertices $(0,0)$, (a,b), (c,d), $(a+c,b+d)$? (You may assume that the parallelogram is an honest-to-goodness parallelogram.) **Explain very carefully what you are doing.**