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Quiz for March 28, 2005

1. Let $\zeta = e^{\frac{2\pi i}{7}}$ and let K be the field $\mathbb{Q}[\zeta]$. Find an element u_1 in K with $\dim_{\mathbb{Q}} \mathbb{Q}[u_1] = 2$.

ANSWER: Let $u_1 = \zeta + \zeta^2 + \zeta^4$. Observe that

$$u_1^2 = \zeta^2 + 2\zeta^3 + 2\zeta^5 + \zeta^4 + 2\zeta^6 + \zeta.$$

It follows that

$$u_1^2 + u_1 = 2(\zeta + \zeta^2 + \zeta^3 + \zeta^4 + \zeta^5 + \zeta^6) = -2.$$

So, u_1 is a root of the polynomial

$$f(x) = x^2 + x + 2.$$

It is easy to see that $f(1)$, $f(-1)$, $f(2)$, and $f(-2)$ are all non-zero. We conclude that $f(x)$ has no linear factors in $\mathbb{Q}[x]$; and therefore, $f(x)$ is irreducible in $\mathbb{Q}[x]$; and $\dim_{\mathbb{Q}} \mathbb{Q}[u_1] = 2$.