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Quiz for March 16, 2005

1. Prove that a regular 7-gon is not constructible.

ANSWER: A regular 7-gon is constructible if and only if $\zeta = e^{\frac{2\pi i}{7}}$ is constructible. It is clear that ζ is a root of $x^7 - 1$. It is also clear that $x^7 - 1 = (x-1)g(x)$, where

$$g(x) = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1.$$

We know that ζ is not a root of $x - 1$; so, ζ is a root of $g(x)$. We also proved that $g(x)$ is irreducible because 6 is equal to $p-1$, where p is the prime integer 7. (Our proof consisted of viewing $g(x) = f(x-1)$ for some polynomial $f(y)$. We used the Eisentstein criteria with $p = 7$ to show that $f(y)$ is irreducible; hence, $g(x)$ is also irreducible.) At any rate, $g(x)$ is the minimal polynomial of ζ. It follows that $\dim_{\mathbb{Q}} \mathbb{Q}[\zeta] = 6$. We proved that if u is a constructible number then $\dim_{\mathbb{Q}} \mathbb{Q}[u] = 2^n$, for some integer n. We know that 6 is not a power of 2 and therefore we conclude that ζ is not constructible.