PRINT Your Name:_____________________________________

Quiz for February 4, 2005

Definition. Let I be an ideal of the ring R, with $I \neq R$. The ideal I is a *prime ideal* of R if, whenever a and b are in R with $ab \in I$, then $a \in I$ or $b \in I$.

Definition. Let I be an ideal of the ring R, with $I \neq R$. The ideal I is a *maximal ideal* of R if R is the only ideal of R which properly contains I.

1. Prove that every maximal ideal is a prime ideal.

ANSWER: Let I be a maximal ideal of the ring R.

Quickest Proof. The ideal I is a maximal ideal, so R/I is a field. It follows that R/I is a domain. It now follows that I is a prime ideal in R.

A direct Proof. Suppose a and b are in R with $ab \in I$ and $a \notin I$. We will prove that b must be in I. The hypothesis that I is a maximal ideal tells us that the ideal (I, a) must be the entire ring; hence, there exist $x \in I$ and $r \in R$ with $1 = x + ra$. Multiply both sides of the equation by b to see that $b = xb + rab \in I$.