PRINT Your Name:____________________________________

Quiz for February 25, 2005

1. Let r be an irreducible element in the Principal Ideal Domain R. Prove that (r) is a prime ideal of R.

ANSWER: I first prove that (r) is a maximal ideal. Suppose I is an ideal of R with $(r) \subsetneq I \subseteq R$. I will show that $I = R$. The ring R is a PID, so $I = (s)$ for some s in R. We know that $r \in (r) \subseteq I = (s)$; so, $r = st$ for some $t \in R$. The ideal I is not equal to (r); hence, t is not a unit. But the element R of r is irreducible; hence, s or t is a unit. We conclude that s is a unit, and $I = R$. We have established that (r) is a maximal ideal. Of course, every maximal ideal in any ring is prime. Indeed, if M is a maximal ideal in R, then $\frac{R}{M}$ is a field. It follows that $\frac{R}{M}$ is a domain; and therefore, M is a prime ideal.