The answer is correct!

Let $u = \sqrt[3]{2}$. In class we calculated that the inverse of $1 + 2u + 3u^2$ in $\mathbb{Q}[u]$ is

$$\frac{1}{801(9)}(9 + (27u + 450)(3u - 2)).$$

Of course, this answer may be cleaned up to become:

$$\frac{1}{801(9)}(81u^2 + 1296u - 891).$$

We now check our answer. We notice that

$$(1+2u+3u^2)\frac{1}{801(9)}(81u^2+1296u-891)$$
$$=\frac{1}{801(9)}\begin{cases} +81u^2+1296u-891\\ +2592u^2-1782u+324\\ -2673u^2+486u+7776 \end{cases} = \frac{7209}{801(9)} = 1. \checkmark$$