Homework Problems   Math 547   April 15, 2005

1. Let $K = \mathbb{Q}[\sqrt{2}, \sqrt{3}, \sqrt{5}]$. Find $u \in K$ with $K = \mathbb{Q}[u]$. Prove that your u works.

2. Let H be the subgroup $\langle (1, 2, 3, 4) \rangle$ of S_4. Let S_4/H be the set of left cosets of H in S_4. Let H act on S_4/H by left translation. In other words, if h is in H and gH is a left coset of H in S_4 (i.e., $g \in S_4$), then h sends gH to the left coset hgH.

   (a) Find the orbit of each element of S_4/H.

   (b) Find the normalizer of H in S_4. Recall that the normalizer of H in S_4 is
   $$N_{S_4}(H) = \{g \in S_4 \mid gHg^{-1} = H\}.$$