Homework Problems Math 547 March 19, 2005

1. Let $\zeta = e^{\frac{2\pi i}{7}}$ and let K be the field $\mathbb{Q}[\zeta]$. Find elements u_1 and u_2 in K with $\dim_{\mathbb{Q}} \mathbb{Q}[u_1] = 2$ and $\dim_{\mathbb{Q}} \mathbb{Q}[u_2] = 3$.

2. Let $\zeta = e^{\frac{2\pi i}{17}}$ and let K be the field $\mathbb{Q}[\zeta]$. Find an elements u_1, u_2, u_3, in K with $\dim_{\mathbb{Q}} \mathbb{Q}[u_1] = 2$, $\dim_{\mathbb{Q}[u_1]} \mathbb{Q}[u_1, u_2] = 2$, and $\dim_{\mathbb{Q}[u_1,u_2]} \mathbb{Q}[u_1, u_2, u_3] = 2$. (When you complete this problem, you will have shown that a regular 17-gon is constructible using ruler and compass.