## Math 547,   Exam 1,   Spring , 2005

The exam is worth 50 points.

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Take enough space for each problem. Turn in your solutions in the order: problem 1, problem 2, ... ; although, by using enough paper, you can do the problems in any order that suits you.

I will e-mail your grade to you.

I will post the solutions on my website later today.

1. (7 points) Let

   $S=\{\frac{a}{b}\in\mathbb{Q}|$ either  $a=0$ , or  $a$  and  $b$  are relatively prime integers and  $3$  does not divide  $b\,\}$.

   Is  $S$  a subring of  $\mathbb{Q}$? Explain.

2. (7 points) Let

   $S=\{\frac{a}{b}\in\mathbb{Q}|$ either  $a=0$ , or  $a$  and  $b$  are relatively prime integers and  $9$  does not divide  $b\,\}$.

   Is  $S$  a subring of  $\mathbb{Q}$? Explain.

3. (7 points) Let  $\phi\colon \mathbb{Z}[x] \to \mathbb{Z}[x]$  be the function which is given by $\phi(f(x)) = (f(x))^2$. Is  $\phi$  a ring homomorphism? Explain.

4. (7 points) Let  $\phi\colon \frac{\mathbb{Z}}{(2)}[x] \to \frac{\mathbb{Z}}{(2)}[x]$  be the function which is given by $\phi(f(x)) = (f(x))^2$. Is  $\phi$  a ring homomorphism? Explain.

5. (7 points) Let  $\phi\colon \mathbb{Q}[x] \to \mathbb{C}$  be the function which is given by  $\phi(f(x)) = f(\sqrt{2})$. All of us know that this function is a ring homomorphism; you do not have to show me a proof. What is the kernel of  $\phi$? Prove your answer.

6. (8 points) Let  $\phi\colon R \to S$  be a ring homomorphism.
   (a)  Prove that  $\phi(0) = 0$.
   (b)  Prove that  $\phi$  is one-to-one if and only if  $\ker \phi = \{0\}$.

7. (7 points) Let  $M$  be an ideal of the ring  $R$. Suppose that  $M \neq R$  and that  $R$  is the only ideal of  $R$  which properly contains  $M$. Prove that  $\frac{R}{M}$  is a field.