**Math 546, Exam 3, Summer 546**

PRINT Your Name:_____

There are 10 problems on 5 pages. Each problem is worth 5 points.

I will put your exam outside my office door after I have graded it. You may pick it up any time before class on Monday. If I know your e-mail address, I will e-mail your score on Exam 3 to you.

1. Define "cyclic group". Use complete sentences.

2. Define "generator of a group". Use complete sentences.

3. State and prove Lagrange's Theorem.

4. Prove that every subgroup of $(\mathbb{Z}, +)$ is cyclic. I want a complete proof. "We did this in class" and "This follows from a Theorem we proved in class" are not good enough.

5. Let $(G, *)$ be an abelian group. Prove that the set

$$S = \{g \in G \mid g * g = \mathrm{id}\}$$

is a subgroup of $G$.

6. Let $G$ be the group $D_3$. (a) LIST the elements of the set

$$S = \{g \in G \mid g * g = \mathrm{id}\}.$$

(b) Is $S$ a subgroup of $G$? Justify your answer to (b).

7. Let $G$ be the group $U_2 \times U_4$. (a) LIST the elements of the set

$$S = \{g \in G \mid g * g = \mathrm{id}\}.$$

(b) Is $S$ a subgroup of $G$? Justify your answer to (b).

8. Give an example of an abelian, non-cyclic, group of order 16. I do not need to see many details.

9. Give an example of a non-abelian group of order 16. I do not need to see many details.

10. Find all of the subgroups of $(\mathbb{Z}_{12}, +)$. Be sure to tell me why you know that you have all of the subgroups.