**Math 546, Exam 1, Summer 2002**

PRINT Your Name:________________________________

There are 8 problems on 5 pages. Problems 1 and 2 are worth 7 points each. Each of the other problems is worth 6 points.

1. Define "group". Use complete sentences.

2. Define "subgroup". Use complete sentences.

3. Define $*$ on $\mathbb{Q} \setminus \{0\}$ by $a * b = \frac{a}{b}$. Is $(\mathbb{Q} \setminus \{0\}, *)$ a group? Why or why not?

4. Recall that $\mathrm{GL}_2(\mathbb{R})$ represents the group of invertible $2 \times 2$ matrices with real number entries. The operation in $\mathrm{GL}_2(\mathbb{R})$ is matrix multiplication. The matrix

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

is an element of $\mathrm{GL}_2(\mathbb{R})$. What is $A$'s inverse?

5. Let $T = \mathbb{R} \setminus \{-2\}$. Define $*$ on $T$ by $a * b = ab + 2a + 2b + 2$. Proof that $(T, *)$ is a group.

6. Recall that $D_3$ is the smallest subgroup of the group of rigid motions which contains $\rho$ and $\sigma$, where $\rho$ is rotation counter clockwise by $120°$ fixing the origin and $\sigma$ is reflection of the $xy$ plane across the $x$ axis. List 4 subgroups of $D_3$ in addition to $D_3$ and $\{\mathrm{id}\}$. (I do not need to see any details.)

7. The Dihedral group $D_4$ consists of 8 elements $\mathrm{id}$, $\rho$, $\rho^2$, $\rho^3$, $\sigma$, $\sigma\rho$, $\sigma\rho^2$, and $\sigma\rho^3$. In class we calculated that $\rho\sigma = \sigma\rho^3$, $\rho^4 = \mathrm{id}$, and $\sigma^2 = \mathrm{id}$. Express $\rho^2\sigma\rho\sigma$ in the form $\sigma^i\rho^j$ for some integers $i$ and $j$, with $0 \le i \le 1$, and $0 \le j \le 3$.

8. Consider $L = \{n \in \mathbb{Z} \mid n \le 7\}$. For $a$ and $b$ in $L$, define $a * b = \min\{a, b\}$. Does $(L, *)$ have an identity element? If yes, what is it and why does it work? If no, why not? (I know that $(L, *)$ is not a group. You do not have to show that, but you do have to answer my question.)