PRINT Your Name:____________________________________

Quiz for March 4, 2010

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.**

Suppose that H is a subgroup of the group G with the property that ghg^{-1} is in H for all $g \in G$ and h in H. Let a, b, and c be elements of G with $aH = bH$, prove that $acH = bcH$.

ANSWER: We are told that there exists $h_0 \in H$ with $a = bh_0$.

We first show that $acH \subseteq bcH$. Take an arbitrary element ach of acH for some h in H. We see that

$$ach = bh_0ch = bc(c^{-1}h_0c)h.$$

The hypothesis ensures that $c^{-1}h_0c$ is an element of H. The set H is a group; so H is closed and $(c^{-1}h_0c)h \in H$. Thus, ach is equal to bc times an element of H and ach is in the left coset bcH.

Now we show that $bcH \subseteq acH$. Take an arbitrary element bch of bcH for some h in H. We see that

$$bch = ah_0^{-1}ch = ac(c^{-1}h_0^{-1}c)h.$$

The hypothesis ensures that $c^{-1}h_0^{-1}c$ is an element of H. The set H is a group; so H is closed and $(c^{-1}h_0^{-1}c)h \in H$. Thus, bch is equal to ac times an element of H and bch is in the left coset acH.