PRINT Your Name:__________________________________

Quiz for March 18, 2010

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.**

Let $\varphi\colon G_1 \to G_2$ and $\theta\colon G_2 \to G_3$ be group homomorphisms.

(1) Prove that the composition $\theta \circ \varphi$, from G_1 to G_3, is a group homomorphism.

(2) Prove that $\ker(\varphi) \subseteq \ker(\theta \circ \varphi)$.

ANSWER:

(1) Let g and g' be elements of G_1. We compute

$$(\theta \circ \varphi)(gg') = \theta(\varphi(gg')) = \theta(\varphi(g)\varphi(g')) = \theta(\varphi(g))\theta(\varphi(g')) = (\theta \circ \varphi)(g)(\theta \circ \varphi)(g).$$

The first equality is due to the definition of composition. The second equality is due to the hypothesis that φ is a homomorphism. The third equality is due to the hypothesis that θ is a homomorphism. The final equality is the definition of composition.

(2) Take g in $\ker \varphi$. We see that

$$(\theta \circ \varphi)(g) = \theta(\varphi(g)) = \theta(\mathrm{id}) = \mathrm{id}.$$

The first equality is the definition of composition. The second equality holds because g is in the kernel of φ. A group homomorphism always carries the identity element of the domain to the identity element of the target. (You proved this result in a different homework problem.) Thus, $(\theta \circ \varphi)(g) = \mathrm{id}$ and g is in the kernel of $\theta \circ \varphi$.