PRINT Your Name:____________________________________

Quiz for February 23, 2010

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.**

Let G be an Abelian group and let H be the subset

$$H = \{g \in G \mid g^2 = \mathrm{id}\}$$

of G. Does H have to be a subgroup of G? If yes, then prove the claim. If no, then give an example.

ANSWER: YES, H is always a subgroup of G as we prove below.

Closure. Take g_1 and g_2 in H. We see that

$$(g_1 g_2)(g_1 g_2) = (g_1 g_1)(g_2 g_2) = \mathrm{id}.$$

(The first equality is due to the fact that G is an Abelian group. The second equality is due to the fact that g_1 and g_2 are both in H.) Thus, the product $g_1 g_2$ is also in H.

Inverses. Take $g \in H$. It follows that $g^2 = \mathrm{id}$; thus g is the inverse of g and the inverse of g (which is g itself) is also in H.

Identity. The identity element squares to the identity element; thus, $\mathrm{id} \in H$.