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Quiz for September 15, 2004

Let g be an element of the group G and let

$$S = \{n \in \mathbb{Z} \mid g^n = \mathrm{id}\}.$$

(In other words, S is the set of integers n such that g^n is equal to the identity of G.) Prove that S is a subgroup of $(\mathbb{Z}, +)$.

ANSWER:

Closure: Take n, m from S. We must show that $n + m$ is in S. Well, $g^{n+m} = g^n g^m = (\mathrm{id})(\mathrm{id}) = \mathrm{id}$. We conclude that $n + m \in S$.

Identity: The integer 0 is in S because g^0 is defined to be the identity element of G.

Inverses: Take $n \in S$. So, $g^n = \mathrm{id}$. Multiply both sides by $(g^{-1})^n = g^{-n}$ to see that $\mathrm{id} = g^{-n}$. We conclude that $-n \in S$.