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Quiz for August 25, 2004

Consider the operation $*$ on $\mathbb{Z}$ which is given by $a * b = \max\{a, b\}$. Is $(\mathbb{Z}, *)$ a group? Why or why not?

ANSWER:

$(\mathbb{Z}, *)$ is NOT a group. There is no identity element. We prove this statement by contradiction. Assume id, in $(\mathbb{Z}, *)$, is an identity element, then

$$\mathrm{id} * (\mathrm{id} - 1) = \max\{\mathrm{id}, \mathrm{id} - 1\} = \mathrm{id}.$$

On the other hand, id is the identity element of $(\mathbb{Z}, *)$, so $\mathrm{id} * (\mathrm{id} - 1) = \mathrm{id} - 1$. Thus,

$$\mathrm{id} = \mathrm{id} - 1,$$

and $0 = -1$. This is impossible. Thus, $(\mathbb{Z}, *)$ does not have an identity element.