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Quiz for April 1, 2004

Let S be a set, and let a be a fixed element of S. Prove that

$$H = \{\sigma \in \mathrm{Sym}(S) \mid \sigma(a) = a\}$$

is a subgroup of $\mathrm{Sym}(S)$.

ANSWER: Closure: If h_1 and h_2 are in H, then $h_1 \circ h_2 \in \mathrm{Sym}(S)$. Furthermore, $h_1 \circ h_2(a) = h_1(h_2(a)) = h_1(a)$ since $h_2 \in H$, and $h_1(a) = a$ since $h_1 \in H$. We conclude that $h_1 \circ h_2$ is in H.
Inverses: Take $h \in H$. It follows that $h(a) = a$. Apply the element h^{-1} of $\mathrm{Sym}(S)$ to both sides to see that $a = h^{-1}(a)$. Therefore, h^{-1} is in H.
H is non-empty because the identity function is in H.
We have established that H is a subfgroup of $\mathrm{Sym}(S)$.