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Quiz for February 5, 2004

Let G be a group. Prove that the center of G is a subgroup of G. (You probably have to tell me what the center of G is.)

ANSWER:

The *center* of the group G is the set

$$Z = \{x \in G \mid xg = gx \text{ for all } g \in G\}.$$

(In other words, the center of G is the set which consists of all elements of G which commute with every element of G.)

The set Z is closed. Suppose x and y are in Z, we must show that xy is in Z. Let g be an arbitrary element of G. We must show that xy commutes with g. Well, $xyg = xgy$ because $y \in Z$ and $xgy = gxy$ because $x \in Z$. Thus, $(xy)g = g(xy)$, and $xy \in Z$.

The set Z is non-empty because the identity element of G is in Z.

The inverse axiom is satisfied. Let x be an element of Z. We know that x has an inverse, called x^{-1}, in G. We must show that x^{-1} is in Z. We must show that x^{-1} commutes with every element of G. Let g be an arbitrary element of G. We know that $xg = gx$ (because $x \in Z$). Multiply both sides of this equation on the left by x^{-1} to get $g = x^{-1}gx$. Multiply both sides of this equation on the right by x^{-1} to get $gx^{-1} = x^{-1}g$. We conclude that $x^{-1} \in Z$.

We proved the following result in class.

Proposition. *Let H be a non-empty subset of the group $(G, *)$. Suppose H is closed under $*$. Suppose, also, that whenever $h \in H$, then the inverse of h in G is also an element of H. Then H is a subgroup of G.*

Apply the Proposition to conclude that Z is a subgroup of G.