PRINT Your Name:________________________________

Quiz for February 24, 2004

Let G be a a cyclic group and let a and b be elements of G such that the equations $a = x^2$ and $b = x^2$ have no solution in G. Prove that $ab = x^2$ does have a solution in G.

ANSWER:

Let g be a generator of G. The hypothesis $a = x^2$ has no solution in G tells us that a must equal g^n for some odd integer n. In a similar manner, we see that $b = g^m$ for some odd integer m. We see that $ab = g^{n+m}$; furthermore, we know that $n + m$ is even. So, $n + m = 2p$ for some integer p; hence, $(g^p)^2 = ab$.