PRINT Your Name:__________________________________

Quiz for February 10, 2004

Let G be the group of rational numbers, under addition, and let H and K be subgroups of G. Assume that $H \neq \{0\}$ and $K \neq \{0\}$. Prove that $H \cap K \neq \{0\}$.

ANSWER: The hypothesis ensures that there is a number $\frac{a}{b}$ in H with a and b positive integers. In a similar way we see that there is a number $\frac{c}{d}$ in K, with c and d positive integers. Observe that ac is a positive integer in $H \cap K$. Indeed, the group H is closed and $\frac{a}{b} \in H$. Add $\frac{a}{b}$ to itself bc times to see that ac is also in H. The group K is closed and $\frac{c}{d} \in K$. Add $\frac{c}{d}$ to itself ad times to see that ac is in K. We conclude that ac is in $H \cap K$.