PRINT Your Name:____________________________________

Quiz for November 8, 2004

Let $\varphi\colon G \to G'$ be a group homomorphism. Consider $\bar{\varphi}\colon \frac{G}{\ker\varphi} \to \operatorname{im}\varphi$, which is given by $\bar{\varphi}(g\ker\varphi) = \varphi(g)$.

(a) Prove that $\bar{\varphi}$ is a **FUNCTION**. That is, if $g_1\ker\varphi$ and $g_2\ker\varphi$ are equal cosets, then prove that $\bar{\varphi}(g_1\ker\varphi) = \bar{\varphi}(g_2\ker\varphi)$.

(b) Prove that the function $\bar{\varphi}$ is one-to-one.

ANSWER: (a) If the cosets $g_1\ker\varphi$ and $g_2\ker\varphi$ are equal, then $g_1 = g_2k$ for some element k of $\ker\varphi$. We see that

$$\bar{\varphi}(g_1\ker\varphi) = \varphi(g_1) = \varphi(g_2k) = \varphi(g_2)\varphi(k) = \varphi(g_2)\mathrm{id} = \varphi(g_2) = \bar{\varphi}(g_2\ker\varphi).$$

(b) Take cosets $g_1\ker\varphi$ and $g_2\ker\varphi$ in $\frac{G}{\ker\varphi}$ with $\bar{\varphi}(g_1\ker\varphi) = \bar{\varphi}(g_2\ker\varphi)$. It follows that $\varphi(g_1) = \varphi(g_2)$. Multiply both sides by $[\varphi(g_2)]^{-1}$ to see that $\varphi(g_1)[\varphi(g_2)]^{-1} = \mathrm{id}$. Observe further that

$$\varphi(g_1{g_2}^{-1}) = \varphi(g_1)\varphi({g_2}^{-1}) = \varphi(g_1)[\varphi(g_2)]^{-1} = \mathrm{id}.$$

Thus, $g_1{g_2}^{-1} \in \ker\phi$ and the cosets $g_1\ker\varphi$ and $g_2\ker\varphi$ are equal.