PRINT Your Name:____________________________________

Quiz for November 15, 2004

Suppose m and n are relatively prime non-zero integers. Prove that the groups $\frac{\mathbb{Z}}{mn\mathbb{Z}}$ and $\frac{\mathbb{Z}}{m\mathbb{Z}} \times \frac{\mathbb{Z}}{n\mathbb{Z}}$ are isomorphic.

ANSWER: A long time ago we proved

Theorem. *Let a and b be elements of finite order in the group $(G, *)$. Suppose $a * b = b * a$ and that the order of a is relatively prime to the order of b. Then the order of $a * b$ is equal to the order of a times the order of b.*

Apply the above theorem to the elements $a = (1, 0)$ and $b = (0, 1)$ of the group $\frac{\mathbb{Z}}{m\mathbb{Z}} \times \frac{\mathbb{Z}}{n\mathbb{Z}}$ in order to conclude that $(1, 1)$ has order nm; and therefore, $\frac{\mathbb{Z}}{m\mathbb{Z}} \times \frac{\mathbb{Z}}{n\mathbb{Z}}$ is a cyclic group of order nm. The group $\frac{\mathbb{Z}}{mn\mathbb{Z}}$ is also a cyclic group of order mn. We have proven that any pair of cyclic groups of the same order are isomorphic to one another.