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Quiz for October 20, 2004

Recall that $\mathbb{Z}_n^\times$ represents the group of cosets

$$\{m + n\mathbb{Z} \mid m \in \mathbb{Z} \text{ with } m \text{ and } n \text{ relatively prime}\},$$

under the operation of coset multiplication:

$$(m_1 + n\mathbb{Z})(m_2 + n\mathbb{Z}) = m_1 m_2 + n\mathbb{Z}.$$

Find all cyclic subgroups of $\mathbb{Z}_9^\times$.

ANSWER: I am going to write $\bar{m}$ instead of $m + 9\mathbb{Z}$. The elements of $\mathbb{Z}_9^\times$ are $\bar{1}, \bar{2}, \bar{4}, \bar{5}, \bar{7}, \bar{8}$. We see that

$$\begin{aligned}
&<\bar{1}> = \{\bar{1}\} \\
&<\bar{2}> = <\bar{5}> = \mathbb{Z}_9^\times \\
&<\bar{4}> = <\bar{7}> = \{\bar{4}, \bar{7}, \bar{1}\} \\
&<\bar{8}> = \{\bar{1}, \bar{8}\}.
\end{aligned}$$