PRINT Your Name:____________________________________

Quiz for October 13, 2004

Prove that $\frac{\mathbb{Z}\times\mathbb{Z}}{<(1,1)>}$ is an infinite cyclic group.

ANSWER: Let H be the subgroup $<(1,1)>$ of the group $G = \mathbb{Z}\times\mathbb{Z}$. We see that the cosets of H in G are

$$\begin{array}{l} \qquad\qquad\vdots \\ (-2,0)+H = \{\dots(-3,-1),(-2,0),(-1,1),(0,2),\dots\} \\ (-1,0)+H = \{\dots(-2,-1),(-1,0),(0,1),(1,2),\dots\} \\ (0,0)+H = \{\dots(-1,-1),(0,0),(1,1),(2,2),\dots\} \\ (1,0)+H = \{\dots(0,-1),(1,0),(2,1),(3,2),\dots\} \\ (2,0)+H = \{\dots(1,-1),(2,0),(3,1),(4,2),\dots\} \\ \qquad\qquad\vdots \end{array}$$

Be sure to notice that each element of G is in exactly one of the left cosets in my list. The group $\frac{G}{H}$ consists of the above set of cosets under the operation $[(a,b)+H]+[(c,d)+H] = (a+c,b+d)+H$. We see that $\frac{G}{H}$ is a cyclic group with generator $g = (1,0)+H$ because every element of $\frac{G}{H}$ is equal to ng for some integer n. P. S. The other generator for $\frac{G}{H}$ is the inverse of g, namely, $-g = (-1,0)+H$.