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Quiz for October 1, 2004

Suppose that H is a subgroup of the group G with the property that ghg^{-1} is in H for all $g \in G$ and h in H. Let a, b, and c be elements of G with $aH = bH$, prove that $acH = bcH$.

ANSWER: The hypothesis that $aH = bH$ tells us that there is an element h_1 in H with $a = bh_1$.

$acH \subseteq bcH$: Take a typical element of acH, say ach, where $h \in H$. Observe that

$$ach = bh_1ch = bcc^{-1}h_1ch = bc(c^{-1}h_1c)h \in bcH.$$

The element inside the parentheses is in H because of the hypothesis; therefore, $(c^{-1}h_1c)h$ is in H by closure.

$bcH \subseteq acH$: Take a typical element of bcH, say bch, where $h \in H$. Observe that

$$bch = ah_1^{-1}ch = acc^{-1}h_1^{-1}ch = ac(c^{-1}h_1^{-1}c)h \in acH.$$