PRINT Your Name:________________________________

Quiz for November 10, 2011

Let $G = D_4$ and $H = <\rho^2>$. We know that H is a normal subgroup of G; so, the factor group $\frac{G}{H}$ exists and makes sense. How many elements are in $\frac{G}{H}$? What is the multiplication table for $\frac{G}{H}$? (If you can describe the multiplication in $\frac{G}{H}$ by using words and not actually writing down the multiplication table that would make a fine answer.) Be sure to justify your answers.

Answer: The group G has 8 elements and the subgroup H has two elements. When we proved Lagrange's Theorem, we saw that the number of elements in G is equal to the number of elements in H times the number of left cosets of H in G. So there are $8/2 = 4$ left cosets of H in G. The elements of $\frac{G}{H}$ are the left cosets of H in G. Thus $\frac{G}{H}$ has 4 elements. It is easy to see that the elements of $\frac{G}{H}$ are $\mathrm{id}H$, σH, $\sigma\rho H$, ρH; further, each element squares to $\mathrm{id}H$ and the product of two of the non-identity elements is the third non-identity element. Thus, $\frac{G}{H}$ is a Klein 4-group.