Consider the function $\varphi : GL_2(\mathbb{R}) \to (\mathbb{R}, +)$ which is given by $\varphi\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d$. Is φ a group homomorphism? Give a proof or a counterexample.

Recall $GL_2(\mathbb{R})$ is the group of invertible 2x2 matrices under multiplication.

φ is not a homomorphism. We proved that a homomorphism always send the identity element to the identity element. The identity element of $GL_2(\mathbb{R})$ is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The identity element of $(\mathbb{R}, +)$ is 0 but

$$\varphi\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 2 \neq 0.$$