PRINT Your Name:__________________________________

Quiz for September 29, 2011

Give an example of a group G and elements a and b in G such that a and b each have finite order but ab does not have finite order.

Answer: Example 1. Let $G = \text{Sym}(\mathbb{C})$. We showed in class that $\text{rot}_\varphi \circ \text{refl}_0 = \text{refl}_{\varphi/2}$. It follows that $\text{rot}_\varphi = \text{refl}_{\varphi/2} \circ \text{refl}_0$. The functions $\text{refl}_{\varphi/2}$ and refl_0 both have order 2; but we can make rot_φ have any order. In particular, if $\varphi = 1$ radian, then $\text{rot}_\varphi^n = \text{rot}_n$ and rot_φ has infinite order. Indeed, rotation by n radians never is the identity map because n is never an integer multiple of 2π since π is irrational. Take $a = \text{refl}_{\varphi/2}$ and $b = \text{refl}_0$. We have shown that a and b both have order 2, but ab has infinite order.

Example 2. Again take $G = \text{Sym}(\mathbb{C})$. Let a be reflection across the line $x = 0$ and b be reflection across the line $x = 1$. Again a and b have order 2. Convince yourself that ab is translation to the left by 2. It is clear that translation has infinite order.