PRINT Your Name:________________________________

Quiz for September 22, 2011

Let σ and ρ be the following two elements of $\mathrm{Sym}(\{1,2,3,4\})$. The function ρ has the behavior: $\rho(1)=2$, $\rho(2)=3$, $\rho(3)=4$, $\rho(4)=1$. The function σ has the following behavior: $\sigma(1)=1$, $\sigma(2)=4$, $\sigma(3)=3$, and $\sigma(4)=2$. Let H be the smallest subgroup of $\mathrm{Sym}(\{1,2,3,4\})$ which contains σ and ρ.

List 6 **subgroups of** H **in addition to** H **and** $\{\mathbf{id}\}$**.** Justify your answer.

Recall that you already know that $\rho^4=\sigma^2=\mathrm{id}$ and $\sigma\rho=\rho^3\sigma$. You also know that H has 8 distinct elements and each of these elements can be written in the form $\rho^i\sigma^j$ with $i\in\{0,1,2,3\}$ and $j\in\{0,1\}$.

Answer: The non-trivial cyclic subgroups of H are $<\rho> = \{\mathrm{id},\rho,\rho^2,\rho^3\}$, $<\rho^2> = \{\rho^2,\mathrm{id}\}$, $<\sigma> = \{\sigma,\mathrm{id}\}$, $<\rho\sigma> = \{\rho\sigma,\mathrm{id}\}$, $<\rho^2\sigma> = \{\rho^2\sigma,\mathrm{id}\}$, $<\rho^3\sigma> = \{\rho^3\sigma,\mathrm{id}\}$. In last week's quiz, we found that the centralizer of σ in H is $\{\mathrm{id},\sigma,\rho^2\sigma,\rho^2\}$. The exact same reasoning as we used last week shows that the centralizer of $\rho\sigma$ in H is $\{\mathrm{id},\rho\sigma,\rho^3\sigma,\rho^2\}$. We have listed 8 subgroups of H.