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Quiz for September 1, 2011

Let G be the group U_9, which consists of all complex numbers z such that $z^9 = 1$.
(a) Which elements g of G can be written in the form h^2 for some $h \in G$?
(b) Which elements g of G can be written in the form h^3 for some $h \in G$?

ANSWER: Let $z_0 = e^{2\pi i/9}$. (If you prefer, $z_0 = \cos(2\pi/9) + i\sin(2\pi/9)$. At any rate the elements of G are $\{z_0^j \mid 0 \le j \le 8\}$.

(a) Every element of G has the form h^2 for some $h \in G$. Indeed, $1 = 1^2$, $z_0 = (z_0^5)^2$, $z_0^2 = (z_0)^2$, $z_0^3 = (z_0^6)^2$, $z_0^4 = (z_0^2)^2$, $z_0^5 = (z_0^7)^2$, $z_0^6 = (z_0^3)^2$, $z_0^7 = (z_0^8)^2$, and $z_0^8 = (z_0^4)^2$.

(b) Only 1, z_0^3 and z_0^6 have the form h^3 for some $h \in G$. Indeed, $1^3 = 1$, $z_0^3 = z_0^3$, $(z_0^2)^3 = z_0^6$, $(z_0^3)^3 = 1$, $(z_0^4)^3 = z_0^3$, $(z_0^5)^3 = z_0^6$, $(z_0^6)^3 = 1$, $(z_0^7)^3 = z_0^3$, and $(z_0^8)^3 = z_0^6$.