Math 546, Exam 3, Spring, 2001

PRINT Your Name:________________________________

There are 9 problems on 5 pages. Problem 1 is worth 12 points. Each of the other problems is worth 11 points.

1. Let $\sigma = (1,2,3)(4,5,6)$ and $\tau = (3,4,5)$ be elements of S_6. Write $\tau\sigma\tau^{-1}$ as the product of disjoint cycles.

2. Is the group $(\mathbb{Z}_{12}^{\times}, \times)$ a cyclic group? Why or why not?

3. Is the group $(\mathbb{Z}_{9}^{\times}, \times)$ a cyclic group? Why or why not?

4. Let A be a set and b be an element of A. Is

$$\{\sigma \in \mathrm{Sym}(A) \mid \sigma(b) = b\}$$

always a subgroup of S_A? If your answer is yes, then PROVE the statement. If your answer is no, then give a COUNTEREXAMPLE.

5. Let A be a set, B be a subset of A, and b be an element of B. Is

$$\{\sigma \in \mathrm{Sym}(A) \mid \sigma(b) \in B\}$$

always a subgroup of S_A? If your answer is yes, then PROVE the statement. If your answer is no, then give a COUNTEREXAMPLE.

6. Let H be a subgroup of the finite group G. Let $x \in G$, and let $[x] = \{y \in G \mid xy^{-1} \in H\}$. Prove that H and $[x]$ have the same number of elements.

7. Let H be the subgroup $\{(1),(12),(13),(23),(123),(132)\}$ of S_4. Let x be the element (124) of S_4, and let $[x] = \{y \in G \mid xy^{-1} \in H\}$. List the elements of $[x]$. (Each element of $[x]$ should appear in your list exactly once.)

8. How many permutations in S_6 have order 3? Explain your answer.

9. Let m and n be integers, and let d be the greatest common divisor of m and n. Prove that there exists integers r and s with $d = rm + sn$.