Math 546, Exam 1, Spring, 2001

PRINT Your Name:_____

There are 8 problems on 4 pages. Problems 1–4 are worth 13 points each. Each of the other problems is worth 12 points.

1. Define "Group".

2. Define "subgroup".

3. True or False (If true, then prove it. If false, then give a counterexample.) If H and K are subgroups of the group G, then the intersection of H and K is a subgroup of G.

4. True or False (If true, then prove it. If false, then give a counterexample.) If H and K are subgroups of the group G, then the union of H and K is a subgroup of G.

5. True or False (If true, then prove it. If false, then give a counterexample.) If H and K are non-zero subgroups of $(\mathbb{Q}, +)$, then the intersection of H and K is non-zero.

6. True or False (If true, then prove it. If false, then give a counterexample.) If H and K are non-zero subgroups of $(\mathbb{R}, +)$, then the intersection of H and K is non-zero.

7. True or False (If true, then prove it. If false, then give a counterexample.) If $(G, *)$ is an abelian group and $H = \{g \in G \mid g * g = e\}$, then H is a subgroup of G.

8. True or False (If true, then prove it. If false, then give a counterexample.) If $(G, *)$ is a group and $H = \{g \in G \mid g * g = e\}$, then H is a subgroup of G.