Math 546, Final Exam, Summer, 1993

Use your own paper. Each problem is worth 10 points.

1. Let $\sigma = (1,3,2)(4,6,5)$ and $\tau = (3,4,5)$ be elements of S_6. Write $\tau\sigma\tau^{-1}$ as the product of disjoint cycles.

2. Let a be a fixed element of the group G. Define the function $\varphi\colon G \to G$ by $\varphi(g) = aga^{-1}$ for all g in G. Prove that φ is a group ISOMORPHISM.

3. TRUE or FALSE: If H_1 and H_2 are subgroups of the group G, then the UNION $H_1 \cup H_2$ is also a subgroup of G. If the statement is true, then PROVE it. If the statement is false, then give a COUNTEREXAMPLE.

4. Let G be a group and let C be the following subset of G:

$$C = \{c \in G \mid cx = xc \text{ for all } x \in G\}.$$

Prove that C is a subgroup of G.

5. What is the order of the element $(1,1)$ in the group $\mathbb{Z}_9 \times \mathbb{Z}_6$? Why?

6. Let H be a subgroup of the group G. Suppose that a and b are elements of G with $aH = bH$. Does $a^{-1}H$ HAVE TO EQUAL $b^{-1}H$? If your answer is "yes", then PROVE the statement. If your answer is "no", then give a counterexample.

7. What is the order of the element $(3,3) + <(1,2)>$ in the group $\dfrac{\mathbb{Z}_4 \times \mathbb{Z}_8}{<(1,2)>}$? Why?

8. Let $\varphi\colon \mathbb{Z} \times \mathbb{Z} \to S_3$ be the function which is defined by

$$\varphi(n,m) = (1,2)^n(1,3)^m.$$

Is φ a homomorphism? Why?

9. Let U be the group of complex numbers of modulus one under multiplication, let $\mathbb{R}$ be the group of Real numbers under addition, and let N be the subgroup

$$N = \{2\pi n \mid n \in Z\}$$

of $\mathbb{R}$. Prove that $\dfrac{\mathbb{R}}{N}$ is isomorphic to U.

10. Let $\varphi\colon G \to G'$ be a group homomorphism. Prove that

$$\overline{\varphi}\colon \frac{G}{\ker\varphi} \to G',$$

given by $\overline{\varphi}\,(a \cdot \ker\varphi) = \varphi(a)$, is a well-defined function.