Math 546, Exam 1, Summer, 1993

Use your own paper. Each problem is worth 10 points.

1. Find the order of $z = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$ in the group $(\mathbb{C}\setminus\{0\},\ \times)$.

2. Recall that U_2 is a cyclic group of order 2 and that U_3 is a cyclic group of order 3. Let G be the direct product group $U_2 \times U_3$.
   (a) Draw the multiplication table for G.
   (b) Is G a cyclic group?

3. Let G be an abelian group and let
$$H = \{a \in G \mid a^2 = e\}.$$
   Prove that H is a subgroup of G.

4. Let H be a subgroup of the group G. Let a be a fixed element of G and let
$$K = \{aha^{-1} \mid h \in H\}.$$
   Prove that K is a subgroup of G.

5. Give an example of a group $(G, *)$ and two elements a and b in G with $(a*b)^2 \neq a^2 * b^2$.