Summer 1993 Final

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① $\tau\sigma\tau^{-1} = (345)(132)(465)(354) = (142)(356)$

② homom $\varphi(gh) = agha^{-1} = aga^{-1}aha^{-1} = \varphi(g)\varphi(h)$ ✓
for all $g, h \in G$

onto If $g \in G$, then $\varphi(a^{-1}ga) = a(a^{-1}ga)a^{-1} = g$ ✓

1-1 If $g \in \ker\varphi$, then $e = \varphi(g) = aga^{-1}$

$\therefore a^{-1}ea = g \quad \therefore e = g$

③ False Take $H_1 = \langle (12) \rangle$ and $H_2 = \langle (23) \rangle$ in $G = S_3$

$H_1 \cup H_2 = \{(1), (12), (23)\}$ which is not a subgroup because it is not closed $(12)(23) = (123)$.

④ closure take c and d in C, we must show that $cd \in C$. If $x \in G$, then $cdx \underset{d \in C}{=} cxd \underset{c \in C}{=} xcd$

thus $cd \in C$

inverses If $c \in C$, we must show $c^{-1} \in C$.
Let x be an element of G.
We know $c \in C$ so $cx = xc$. Thus $c^{-1}cxc^{-1} = c^{-1}xcc^{-1}$.
In other words $xc^{-1} = c^{-1}x$. It follows that $c^{-1} \in C$.

nonempty $e \in C$.

⑤ $(1,1)$ has order 18 because $18 \cdot (1,1) = (18, 18) = (0,0)$
thus the order of $(1,1)$ divides 18 but
$2(1,1)$, $3(1,1)$ $6(1,1)$ $9(1,1)$ are all not $(0,0)$

⑥ No. Take $H = \langle (12) \rangle$ and $G = S_3$. Take $a = (13)$ $b = (123)$

$a^{-1} = (13)$ $b^{-1} = (132)$

$aH = \{(13)\ (13)(12) = (123)\}$
$bH = \{(123)\ (123)(12) = (13)\}$
equal

$a^{-1}H = aH = \{(13), (123)\}$
$b^{-1}H = \{(132), (132)(12) = (23)\}$
not equal