5. Let $T = \mathbb{R} \setminus \{-2\}$. Define $*$ on T by $a * b = ab + 2a + 2b + 2$. Proof that $(T, *)$ is a group.

Closure Take a and b $\in T$. It is clear that $a*b$ is in $\mathbb{R}$. Furthermore if $a*b=-2$, then $ab+2a+2b+2=-2$, so $ab+2a+2b+4=0$, so $(a+2)(b+2)=0$ so either $a=-2$ or $b=-2$. But $a,b \in T$ so neither a nor b is equal to -2. Thus $a*b \neq -2$.

Assoc Take a, b, c in T.

$$(a*b)*c = (ab+2a+2b+2)*c = (ab+2a+2b+2)c + 2(ab+2a+2b+2) + 2c + 2$$
$$= abc + 2ab + 2ac + 2bc + 4a + 4b + 4c + 6$$
$$a*(b*c) = a*(bc+2b+2c+2) = a(bc+2b+2c+2) + 2a + 2(bc+2b+2c+2)$$
$$= abc + 2ab + 2ac + 2bc + 4a + 4b + 4c + 6$$

Thus $(a*b)*c = a*(b*c)$ and $*$ is an associative operation

Identity -1 is the identity element because

$$a*(-1) = a(-1) + 2a + 2(-1) + 2 = a$$ and
$$(-1)*a = (-1)a + 2(-1) + 2a + 2 = a$$

Inverses Let $a \in T$. Observe that $b = \frac{-2a-3}{a+2}$ is also in T since $a \neq -2$ and $b \neq -2$ because otherwise $-2a-3 = -2(a+2)$ so $-3=-4$ and this is not true.

I will show that b is a's inverse.

$$a*b = a\left(\frac{-2a-3}{a+2}\right) + 2\left(\frac{-2a-3}{a+2}\right) + 2a + 2$$
$$= \frac{-2a^2-3a-4a-6}{a+2} + 2a + 2$$
$$= \frac{-(2a^2+7a+6)}{a+2} + 2a + 2 = \frac{-(a+2)(2a+3)}{(a+2)} + 2a + 2$$
$$= -2a - 3 + 2a + 2 = -1$$

and of course $b*a = a*b = -1$.