Math 546, Final Exam, Summer , 2001

PRINT Your Name:________________________________

Get your course grade from **TIPS/VIP** late on Monday or later; or e-mail your e-mail address to me and I will e-mail your grade to you.

There are 20 problems on 8 pages. The exam is worth a total of 100 points. Each problem is worth 5 points.

1. DEFINE *group isomorphism.*

2. DEFINE *generator.*

3. DEFINE *centralizer.*

4. DEFINE *normal subgroup.*

5. STATE Lagrange's Theorem.

6. STATE the lemma from number theory about linear combinations and greatest common divisors.

7. STATE the lemma about the order of the element ab in terms of the order of a and the order of b.

8. Pick one of the statements from problems 5 through 7. Tell me which statement you have chosen. PROVE the statement.

9. What is the order of the element $([4]_6, (12)(34))$ in the group $\mathbb{Z}_6 \times S_4$? Explain your answer.

10. The subgroup $N = \{\text{id}, (12)(34), (13)(24), (14)(23)\}$ of the group S_4 is normal. What is the order of the element $N(1234)$ in the group $\frac{S_4}{N}$? Explain your answer.

11. Let $(\mathbb{R}^{\text{pos}}, \times)$ represent the group of positive real numbers under multiplication. Is $(\mathbb{R}^{\text{pos}}, \times)$ isomorphic to $(\mathbb{R}, +)$? If so, exhibit an isomorphism between the two groups. If not, explain why not.

12. Exhibit two groups of order 25 which are not isomorphic. Explain why the groups are not isomorphic.

13. Consider $\varphi\colon \mathbb{Z}_4 \to \mathbb{Z}_{12}$, which is given by $\varphi([a]_4) = [a]_{12}$. Is φ a function? Explain.

14. Consider $\varphi\colon \mathbb{Z}_{12} \to \mathbb{Z}_4$, which is given by $\varphi([a]_{12}) = [a]_4$. Is φ a function? Explain.

15. How many permutations in S_6 have order 4 . Explain your answer.

16. What is the inverse of $[39]_{83}$ in $(\mathbb{Z}_{83}^{\times}, \times)$. Check your answer.

17. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) All groups of order 5 are isomorphic.

18. Recall that $(2\mathbb{Z}, +)$ is the group of even integers. Prove that the function $\varphi\colon (\mathbb{Z}, +) \to (2\mathbb{Z}, +)$, which is given by $\varphi(n) = 2n$, is a group isomorphism.

19. Let K and N be subgroups of the group G. Let

$$S = \{kn \mid k \in K \text{ and } n \in N\}.$$

If N is a normal subgroup of G, then prove that S is a subgroup of G.

20. Let G be an abelian group. Let $H = \{x^2 \mid x \in G\}$. Prove H is a subgroup of G.