PRINT Your Name:_____

There are 10 problems on 6 pages. Each problem is worth 5 points. The exam is worth 50 points.

1. Are the groups $(\mathbb{Z}_{15}^{\times}, \times)$ and $(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2, +)$ isomorphic? Explain.

$\underbrace{\qquad}_{G}$ $\underbrace{\qquad\qquad}_{G'}$

(NO) $[2]_{15} \in G$ has order 4 since $[2]_{15}^2 = [4]_{15}$ $[2]_{15}^3 = [8]_{15}$ $[2]_{15}^4 = [1]_{15}$.

Every element in G' has order 2 or less.

If $\varphi : G \to G'$ ~~were~~ were an isomorphism, then $\varphi\left([2]_{15}\right)$ would

have to have order 4.

2. Are the groups $(\mathbb{Z}_2 \times \mathbb{Z}_3, +)$ and $(U_6, \times)$ isomorphic? Explain.

(Yes) Each group is a cyclic group of order 6. All cyclic groups of order 6 are isomorphic.

$\mathbb{Z}_2 \times \mathbb{Z}_3$ is generated by $(1,1)$.

U_6 is generated by $\cos \frac{2\pi}{6} + i \sin \frac{2\pi}{6}$.