8. (6 points) This problem has THREE parts. Let a and b be elements of finite order in a group G.

   (a) LIST two hypothesis so that if a and b satisfy these hypotheses, then the order of ab is equal to the order of a times the order of b.

   ① $O(a)$ and $O(b)$ are relatively prime.

   ② $ab = bq$

   (b) Give an EXAMPLE where the FIRST hypothesis holds, but the SECOND hypothesis fails and the conclusion fails.

   Take σ and ρ in D_3 ! $O(\sigma) = 2$ $O(\rho) = 3$ 2 and 3 are relatively prime but $\sigma\rho \neq \rho\sigma$ and $O(\sigma\rho) = 2 \neq 2 \cdot 3$

   (c) Give an EXAMPLE where the SECOND hypothesis holds, but the FIRST hypothesis fails and the conclusion fails.

   Take σ and ρ^2 in D_4 ! $\sigma\rho^2 = \rho^2\sigma$ but $O(\sigma) = O(\rho^2) = 2$ and $O(\sigma\rho^2) = 2 \neq 2 \cdot 2$