546 Exam 2 Summer 2001

27

PRINT Your Name:______________________________

There are 10 problems on 5 pages. Each problem is worth 5 points.

1. Define "cyclic group". Use complete sentences.

The group G is a cyclic group if there exists an element $g \in G$ so that each element of G is equal to g^m for some integer m.

2. Define "center". Use complete sentences.

The center of the group G is

$$Z(G) = \{x \in G \mid xg = gx \text{ for all } g \in G\}.$$

3. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.)
If every proper subgroup H of a group G is abelian, then G is abelian.

False The group D_3 is non-abelian because $\sigma\rho \neq \rho\sigma$. The proper subgroups of D_3 have order 1, 2, or 3 by Lagrange's Theorem. A group of order 2 or 3 is cyclic hence abelian (again by Lagrange's Theorem). The only group of order 1 is $\langle id \rangle$. So every proper subgroup of G is abelian.