(13)

6 4. Let $T=\mathbb{R}\setminus\{1\}$. Define $*$ on T by $a*b=ab-a-b+2$. Proof that $(T,*)$ is a group.

<u>closure</u> Let $a,b\in T$. It is clear that $a*b\in\mathbb{R}$. If $a*b=1$, then

$ab-a-b+2=1$ so $ab-a-b+1=0$ $(a-1)(b-1)=0$ so $a=1$ or $b=1$

But $a,b\in T$. Thus $a,b\neq 1$, and $a*b\in T$

<u>Associativity</u> Let $a,b,c\in T$. We see that

$a*(b*c)=a*(bc-b-c+2)=a(bc-b-c+2)-a-(bc-b-c+2)+$

$(a*b)*c=(ab-a-b+2)*c=(ab-a-b+2)c-(ab-a-b+2)-c+2.$

Both expressions equal $abc-ab-ac-bc+a+b+c$, thus

$a*(b*c)=(a*b)*c.$

<u>Id</u> 2 is the identity element because

$a*2=a2-a-2+2=2a-a=a$

of course, $2*a=a*2$

<u>inverses</u> Let a be in T. I look for a's inverse (call it b) with

$a*b=2$ $ab-a-b+2=2$ $ab-a-b=0$ $b(a-1)=a$ $b=\frac{a}{a-1}$

<u>Notice</u> $a\neq 1$ so $\frac{a}{a-1}\in\mathbb{R}$ also $\frac{a}{a-1}\neq 1$ because otherwise $a=a-1$

$0=-1$

and this is absur.

we check that $\frac{a}{a-1}$ really is the inverse of a:

$$a*\frac{a}{a-1}=a\frac{a}{a-1}-a-\frac{a}{a-1}+2=\frac{a^2-a}{a-1}-a+2=\frac{a(a-1)}{a-1}-a+2$$

$=2$ as desired.

of cours $\frac{a}{a-1}*a=2$ also