

PRINT Your Name: _____

There are 9 problems on 6 pages. The exam is worth 50 points.

1. (5 points) Define “generator”. Use complete sentences.
2. (5 points) Define “centralizer”. Use complete sentences.
3. (6 points) STATE and PROVE Lagrange’s Theorem.
4. (5 points) Is $(\mathbb{Z}_{14}^\times, \times)$ a cyclic group? If so, exhibit a generator. If not, explain why not.
5. (6 points) This problem has TWO parts. Let $(G, +)$ be an abelian group. Let $T_3(G) = \{g \in G \mid g + g + g = 0\}$.
 - (a) Prove $T_3(G)$ is a subgroup of G .
 - (b) Compute $T_3(\mathbb{Z}_6, +)$.
6. (6 points) TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If G is a group and $H = \{x^3 \mid x \in G\}$, then H is a subgroup of G .
7. (5 points) Find the inverse of $[37]_{83}$ in $(\mathbb{Z}_{83}^\times, \times)$.
8. (6 points) This problem has THREE parts. Let a and b be elements of finite order in a group G .
 - (a) LIST two hypothesis so that if a and b satisfy these hypotheses, then the order of ab is equal to the order of a times the order of b .
 - (b) Give an EXAMPLE where the FIRST hypothesis holds, but the SECOND hypothesis fails and the conclusion fails.
 - (c) Give an EXAMPLE where the SECOND hypothesis holds, but the FIRST hypothesis fails and the conclusion fails.
9. (6 points) This problem has TWO parts.
 - (a) LIST the right cosets of $\langle \sigma \rangle$ in D_4 .
 - (b) Let S equal the set of right cosets of $\langle \sigma \rangle$ in D_4 . Is

$$(\langle \sigma \rangle x, \langle \sigma \rangle y) \mapsto \langle \sigma \rangle xy$$

a well-defined FUNCTION from $S \times S$ to S ? EXPLAIN.