Math 546, Spring 2004, Exam 3

PRINT Your Name:________________________________

There are 8 problems on 4 pages. The exam is worth 50 points.

I won't grade your exam until Monday. So don't be surprised if I don't e-mail your grade to you until then.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your exam outside my office after I have graded it. (If you like, I will send you an e-mail when I am finished with it.) You may pick it up any time between then and the next class. **Let me know if you are interested.**

I will post the solutions on my website tonight after the exam is finished.

1. (5 points) Define "order". Use complete sentences.

2. (5 points) List ALL of the generators of $(\mathbb{Z}_8, +)$. No explanation is needed.

3. (5 points) List ALL of the subgroups of $(U_{12}, \times)$. No explanation is needed.

4. (5 points) Is $(\mathbb{Z}_{15}^{\times}, \times)$ a cyclic group? Explain.

5. (5 points) Recall that each element of $\mathbb{C}$ is a point on the complex plane. Give a geometric description of the left cosets of U in $(\mathbb{C} \setminus \{0\}, \times)$.

6. (5 points) PROVE that every subgroup of $(\mathbb{Z}, +)$ is cyclic.

7. (4 points) Let m and n be positive integers and let d be the greatest common divisor of m and n. PROVE that there exist integers r and s with $d = rm + sn$.

8. Let a and b be elements of finite order in the group G.
   (a) (4 points) List two hypotheses (Hypothesis (1) and Hypothesis (2)) with the property that if Hypothesis (1) and Hypothesis (2) both hold, then the order of ab is equal to the order of a times the order of b.
   (b) (4 points) Give an example where Hypothesis (1) holds, Hypothesis (2) fails to hold, and the conclusion also fails to hold.
   (c) (4 points) Give an example where Hypothesis (2) holds, Hypothesis (1) fails to hold, and the conclusion also fails to hold.
   (d) (4 points) Prove the result which you stated in (a).