Math 546 Exam 2 Spring 2004

PRINT Your Name:________________________________

There are 7 problems on 5 pages. Problems 1 and 2 are worth 10 points each. Each of the other problems is worth 6 points.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your exam outside my office after I have graded it. (If you like, I will send you an e-mail when I am finished with it.) You may pick it up any time between then and the next class. **Let me know if you are interested.**

I will post the solutions on my website tonight after the exam is finished.

1. STATE and PROVE Lagrange's Theorem.

2. Let G be a group and g be an element of G.
   (a) Define the *center*, $Z(G)$, of G.
   (b) Define the *centralizer*, $C_G(g)$, of g in G.
   (c) Is it always true that $C_G(g) \subseteq Z(G)$? If yes, prove it. If no, give a counterexample.
   (d) Is it always true that $Z(G) \subseteq C_G(g)$? If yes, prove it. If no, give a counterexample.

3. (Yes or No. If yes, PROVE it. If no, give a COUNTEREXAMPLE.) Let H and K be subgroups of the group G with $H \neq \{\text{id}\}$ and $K \neq \{\text{id}\}$. Is it always true that $H \cap K \neq \{\text{id}\}$?

4. (Yes or No. If yes, PROVE it. If no, give a COUNTEREXAMPLE.) Let G be a group in which every proper subgroup is cyclic. Does the group G have to be cyclic?

5. (Yes or No. If yes, PROVE it. If no, give a COUNTEREXAMPLE.) Let G be a group and let S be the subset $S = \{x \in G \mid x^2 = \text{id}\}$ of G. Is S always a subgroup of G?

6. (Yes or No. If yes, PROVE it. If no, give a COUNTEREXAMPLE.) Let G be an abelian group and let S be the subset $S = \{x \in G \mid x^2 = \text{id}\}$ of G. Is S always a subgroup of G?

7. List the left cosets of the subgroup $H = \{\text{id}, \rho, \rho^2, \rho^3\}$ in the group $G = D_4$. I do not need to see many details.