74 81

19. Let K and N be subgroups of the group G. Let

$$S = \{kn \mid k \in K \text{ and } n \in N\}.$$

If N is a normal subgroup of G, then prove that S is a subgroup of G.

Closure $kn \cdot k_1 n_1 = k k_1 (\underbrace{k_1^{-1} n k_1}_{\text{in } N \text{ because } N \text{ is normal}} n_1) = \text{elt of } K \cdot \text{elt of } N \in S$

inverses Take $kn \in S$ we see that $(kn)^{-1} = n^{-1}k^{-1} = k^{-1}(\underbrace{kn^{-1}k^{-1}}_{\text{in } N \text{ because } N \text{ is normal}}) \in \text{elt of } K \cdot \text{elt of } N \in S$

7

The subset S of the group G is nonempty and is closed under the group operation and the process of forming inverse. Thus S is a subgroup of G.

20. The subgroup $N = \{\text{id}, (12)(34), (13)(24), (14)(23)\}$ of the group S_4 is normal. The factor group $\frac{S_4}{N}$ is isomorphic to which familiar group? Explain your answer.

7

The elements of $\frac{S_4}{N}$ are

$idN = \{id, (12)(34), (13)(24), (14)(23)\}$

$(12)N = \{(12), (34), (1324), (1423)\}$

$(13)N = \{(13), (1234), (24), (1432)\}$

$(23)N = \{(23), (1342), (1243), (14)\}$

$(123)N = \{(123), (134), (243), (142)\}$

$(132)N = \{(132), (234), (124), (143)\}$

It is clear that $\frac{S_4}{N} \to S_3$ is a group isomorphism.

σN with $\sigma \in S_3$ $\longmapsto \sigma$