

PRINT Your Name: _____

Get your course grade from TIPS/VIP late on Tuesday or later.

There are 20 problems on 8 pages. The exam is worth a total of 150 points. Problems 1 through 10 are worth eight points each. Problems 11 through 20 are worth 7 points each.

1. DEFINE group. A set G together with a binary operation $*$: $G \times G \rightarrow G$ is called a group if

a) (closure) $g_1 * g_2 \in G$ for all $g_1, g_2 \in G$

b) (associativity) $g_1 * (g_2 * g_3) = (g_1 * g_2) * g_3$ for all $g_1, g_2, g_3 \in G$

c) (identity) there exists an element $id \in G$ with $id * g = g$ and $g * id = g$ for all $g \in G$

d) (inverse) for each element $g \in G$, there is an element $g^{-1} \in G$ with $g * g^{-1} = id$ and $g^{-1} * g = id$.

2. DEFINE cyclic group. The group G is cyclic if there is an element $g \in G$ so that every element of G is equal to g^n for some integer n .

3. DEFINE the center of a group. The center of the group G is equal to $\{g \in G \mid ag = ga \text{ for all } a \in G\}$.